Concentration Polarization in Cross-Flow Reverse Osmosis

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A dimensionless governing equation was formulated for a cross-flow RO process in which the local variation of concentration polarization was rigorously considered. It was shown in this formulation that the cross-flow RO process could be fully characterized with a single dimensionless parameter. The coupling between permeate flux and concentration polarization was properly solved and a closed-form analytical solution was obtained. This analytical solution enabled us to conveniently investigate concentration polarization in the RO process. The significance of local variation of concentration polarization was demonstrated, and the operations of RO under various conditions were simulated and investigated with the newly developed model.

Introduction

The reverse osmosis (RO) membrane separation process is becoming one important method for separation of solutes from aqueous solutions. Applications of the RO process include desalination of brackish water and sea water, production of ultra-pure water for various industries, treatment of industrial wastes, purification of products in food, chemical, and pharmaceutical industries, and removal of inorganic and organic pollutants from water and wastewater (Weber, 1972; Bhattacharyya et al., 1990). The RO process has shown remarkable advantages over many conventional separation processes (Soltanieh and Gill, 1981). For example, in one of its major applications, the RO process converts sea water to drinking water at a price of one-third of that for the thermal distillation process (Potts et al., 1981). The continuous improvement in membrane manufacturing for higher water permeability and higher salt rejection at a lower price will lead to wider applications of the process in new fields.

The permeate flux of the RO process is a major concern in both system design and operation optimization. The permeate flux in a RO process is initially controlled by the properties of membrane transport. This means that the initial permeate flux of the RO process is determined by the membrane permeability (or resistance) to water. As filtration proceeds, solute rejected by the membrane forms a layer of high concentration near the membrane surface. This phenomenon is called concentration polarization. The formation and development of the concentration polarization layer in the RO process will reduce the effective driving force for water filtration and, therefore, reduce permeate flux. It has been recognized for a long time that concentration polarization is a major limiting factor to permeate flux in many RO membrane processes (Bhattacharyya et al., 1990; Marinas and Urama, 1996; Bhattacharya and Hwang, 1997).

When concentration polarization is insignificant, permeate flux in the RO process can be calculated from membrane transport models. Many models have been developed for membrane transport, such as thermodynamic models, a solution-diffusion model, and a finely porous model. A few excellent review articles about these models are available in the literature (Soltanieh and Gill, 1981; Mason and Lonsdal, 1990). One serious drawback of the existing models is that the parameters in these models are concentration-dependent. A new transport model was developed recently for the RO membrane in which the parameters are concentration-independent (Song, 1999). It has been shown that the new model can correctly predict permeate flux for any operational conditions, as long as concentration polarization is insignificant.

Significant concentration polarization can develop in many RO processes, especially for filtration of high salt concentrations under high pressures. Because the additional osmotic pressure that arises from concentration polarization reduces the effective driving force for water, the flux in such processes falls below that predicted with the membrane trans-

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port models.

The development of the concentration polarization layer in cross-flow filtration is a complex process that is affected by many factors, such as the configuration of the filtration channel, shear rate, solute concentration and diffusivity, operational pressure, and membrane resistance. There are two particular features for concentration polarization in cross-flow RO process. First, as a result of solute rejection and accumulation, the concentration polarization layer grows gradually along the filtration channel. Second, concentration polarization is coupled with permeate flux. On the one hand, the development of concentration polarization is induced by a permeate flux that brings solute to the vicinity of the membrane surface. On the other hand, the development of concentration polarization creates an additional resistance layer to permeate flux. Neither concentration polarization nor permeate flux can be determined independently. These two features make the investigation of concentration polarization in the cross-flow RO process one of the most challenging tasks in this field.

Previous work on concentration polarization in RO process is briefly reviewed. A new model for concentration polarization in the cross-flow RO process is developed in which the local variation of concentration polarization and the coupling between concentration polarization and permeate flux are rigorously handled. With an analytical solution, this model can be conveniently used to predict the local variations of concentration polarization and permeate flux in a RO channel. Permeate fluxes in the cross-flow RO process under different operational conditions are simulated and discussed with the new model.

Previous Work on Concentration Polarization

Concentration polarization in the RO process is a mass-transfer problem controlled by convection and diffusion of the solute. Therefore, theories and methods in other mass-transfer and boundary layer studies are often used in the study of concentration polarization in the RO process (Hwang and Kammermeyer, 1975; De and Bhattacharya, 1997). The concentration polarization layer is commonly treated as a uniform layer over the membrane surface (Kimura, 1995; Niemi and Palosaari, 1993; Marinas and Urama, 1996) so that concentration polarization can be related to permeate flux simply by

$$\frac{c_w - c_p}{c_0 - c_p} = \exp\left(\frac{\bar{v}}{k}\right) \tag{1}$$

where c_w is the solute concentration on the membrane surface, c_0 and c_p are the solute concentrations (mol/m^3) in the feed and the permeate respectively, $\bar{v}$ is the average permeate flux (m/s), and k is the solute mass-transfer coefficient (m/s). k is usually calculated from various correlations between the Sherwood number Sh, Reynolds number R_e, and Schmidt number S_c (Kimura, 1995; Niemi and Palosaari, 1993; Marinas and Urama, 1996), which can be generally expressed as

$$Sh = f(R_e, S_c) \tag{2}$$

Recently, some interesting findings were reported about the mass-transfer coefficient for the RO process. Bhattacharya and Hwang (1997) showed that the coefficient k was not strictly equal to D/δ. The k approaches to D/δ only when the so-called average modified Peclet number approaches zero. De and Bhattacharya (1997) studied the local k along the filtration channel first. Then, the local mass-transfer coefficient was averaged over the entire channel to obtain the mean mass-transfer coefficient.

Though Eqs. 1 and 2 have been used widely in calculating permeate fluxes in RO membrane processes, the drawbacks of this method are obvious. The assumption of a uniform concentration polarization layer along the filtration channel is not suitable for the cross-flow RO process, where the concentration polarization layer develops gradually along the filtration channel. Furthermore, the coupling between concentration polarization and permeate flux makes it impossible to calculate the mass-transfer coefficient k independently. Any attempt that tries to determine k first and then use it to calculate permeate flux cannot be justified in principle.

Marinas and Urama (1996) considered the local variation of concentration polarization in their numerical calculation for permeate flux and salt rejection in a spiral-wound reverse osmosis element. In their study, the element was divided into n equal segments. The wall concentration and permeate flux were assumed to be constant in each segment. The development of concentration polarization along the channel was considered by taking the concentrate of a segment as the feed to the next one. The local variation of wall concentration and permeate flux in the cross-flow RO process were also studied by Bhattacharyya et al. (1990) with a numerical solution of the diffusion-convection equation. Though concentration polarization and permeate flux in the cross-flow RO process can be better studied, these numerical methods might not be very attractive from a designer's point of view because extensive computational effort is needed.

We may conclude from the above review that the current theoretical descriptions of concentration polarization in the cross-flow RO process are rather limited. The assumption of the uniform concentration polarization layer oversimplifies the cross-flow RO process. On the other hand, the numerical methods are unlikely to be widely used because of the extensive computational requirement. A model that is capable of dealing with the local variation of concentration polarization and, at the same time, has a closed-form analytical solution will be most desirable.

Theory Development

Thermodynamic principle of membrane transport

It was demonstrated recently (Song, 1999) that water and solute transports through RO membranes could be well described with the principles of irreversible thermodynamics. In a RO system without concentration polarization, water flux can be calculated from

$$v = \frac{\Delta p - nRTrc_0}{R_m} = \frac{\Delta p - \Delta\pi_0}{R_m} = \frac{\Delta P}{R_m} \tag{3}$$

where Δp is the applied pressure (Pa), n is the number of ions in one solute molecule, R is the gas constant (J/mol·K), T is the absolute temperature (K), r is the solute rejection, c_0 is the feed solute concentration, and R_m is the membrane resistance (Pa·s/m) to pure water. $\Delta\pi_0$ $(=nRTrc_0)$ is the osmotic pressure difference between feed and permeate sides of the membrane (Pa), and ΔP $(=\Delta p-\Delta\pi_0)$ is the effective driving force for permeate (Pa).

The solute rejection r is a performance variable of the RO process that can be determined from (Song, 1999)

$$r=1-\left(\frac{c_w}{c_s}+r\frac{\overline{V}_s}{\overline{V}_w}\right)\frac{1}{S}\frac{c_s^2}{c_w}\left[\frac{\overline{V}_s\Delta p-nRT\ln(1-r)}{\Delta p-nRTc_s r}\right] \tag{4}$$

where S is the selectivity of the membrane. (The selectivity is defined as the ratio of the conductivity of water to the conductivity of solute through the membrane.) $\overline{V}_s$ and $\overline{V}_w$ are the partial molar volumes (m^3/mol) of solute and water, respectively. The solute rejection r can be solved from Eq. 4 by an iteration procedure (Song, 1999).

The model Eq. 3 and Eq. 4 were compared with Pusch's RO experimental data (Pusch, 1977). The results showed that the predicted permeate fluxes agreed with the experimental observations for low salt concentrations where concentration polarization was negligible.

In RO processes where concentration polarization is significantly developed, the additional osmotic pressure arising from the buildup of the concentration polarization layer has to be considered. In order to determine the additional osmotic pressure, the development of concentration polarization in cross-flow RO filtration needs to be deliberated.

Development of concentration polarization

Concentration polarization is induced by permeate flux, because the retained solute is brought to the vicinity of the membrane by permeate flow. On the other hand, the additional osmotic pressure resulted from the concentration polarization layer reduces permeate flux, which in turn reduces concentration polarization. The interplay between permeate flux and concentration polarization becomes more complex in cross-flow filtration where the concentration polarization layer develops gradually along the filtration channel (Song and Elimelech, 1995; Elimelech and Bhattacharjee, 1998). An intrinsic relationship between concentration polarization and permeate flow in cross-flow filtration is developed below.

In order to simplify notation and to facilitate derivation, the concentration of retained solute C is used below. (The actual solute concentration in the polarization layer is c_0+C, with c_0 being the feed solute concentration.) At steady state, the concentration of retained solute in the concentration polarization layer satisfies the following equation (Song and Elimelech, 1995)

$$v(x)C+D\frac{dC}{dy}=0 \tag{5}$$

where x and y are the longitudinal and transverse coordinates (m), respectively, $v(x)$ is the permeate flux at x, and D is the solute diffusion coefficient (m^2/s). The direction of permeate flux here is defined as from the bulk to the membrane, so that $v(x)$ is always positive. Since Eq. 5 is a mass balance relationship for the "retained solute," it is always correct regardless that the solute is completely or partially rejected by the membrane.

Because of steady state, the longitudinal solute flux at any point x along the channel is equal to the total amount of solute rejected by the membrane from the inlet to the location x, that is (Song, 1998)

$$\int_0^\infty \gamma yC\,dy=rc_0\int_0^x v(x')\,dx' \tag{6}$$

where γ is the shear rate (s^{-1}), and x' is the dummy integration variable. Equation 6 will be true as long as the steady state is reached and the rejection r does not change with location.

The solution of Eq. 5 can be written as

$$C=Ae^{-[v(x)/D]y} \tag{7}$$

where A is an integration constant to be determined. From Eqs. 6 and 7, we have

$$A=\frac{rc_0}{D^2\gamma}v(x)^2\int_0^x v(x')\,dx' \tag{8}$$

Substituting Eq. 8 for A in Eq. 7 yields

$$C=\left[\frac{rc_0}{D^2\gamma}v(x)^2\int_0^x v(x')\,dx'\right]e^{-[v(x)/D]y} \tag{9}$$

Let $y=0$ in Eq. 9, one obtains the wall concentration

$$C_w=\frac{rc_0}{D^2\gamma}v(x)^2\int_0^x v(x')\,dx' \tag{10}$$

or

$$\frac{C_w}{c_0}=\frac{r}{D^2\gamma}v(x)^2\int_0^x v(x')\,dx' \tag{11}$$

Equation 11 is a fundamental relationship between concentration polarization and permeate flux in the cross-flow RO process. This equation indicates that the magnitude of concentration polarization is affected by salt rejection, diffusion coefficient, shear rate, and, more importantly, permeate flux.

Osmotic pressure

Additional osmotic pressure can arise on the feed side of the membrane due to concentration polarization. The osmotic pressure of a solution of several electrolytes can be calculated with

$$\pi=\phi RT\left(\sum_i n_i c_i\right) \tag{12}$$

where π is the osmotic pressure (Pa), n_i is the number of ions in electrolyte i, c_i is the concentration of the electrolyte

i, and ϕ is the osmotic coefficient. The osmotic coefficient is a function of electrolyte concentration, and it approaches unity for infinitely dilute solutions. The value of ϕ for single and multicomponent electrolyte solutions can be determined from experiments or calculated using Pitzer equations (Pitzer, 1974, 1979).

For solutions of single electrolyte, one has

$$\pi = n\phi RTc \tag{13}$$

From Eq. 10 and Eq. 13, the additional osmotic pressure, $\Delta\pi_a$ (Pa) due to concentration polarization is determined as

$$\Delta\pi_a = n\phi RT(c - c_0) = n\phi RTC = \frac{n\phi RTrc_0}{D^2\gamma} v(x)^2 \int_0^x v(x')\,dx' \tag{14}$$

General equation for cross-flow RO process

When concentration polarization is significantly developed, the driving force for water through the membrane becomes $\Delta P - \Delta\pi_a$. Therefore, the transport equation for permeate flux through the RO membrane needs to be modified from Eq. 3 to

$$v(x) = \frac{\Delta P - \Delta\pi_a}{R_m} \tag{15}$$

Substituting Eq. 14 for $\Delta\pi_a$ in Eq. 15 and rearranging the result in the general equation for the permeate flux in the cross-flow RO process

$$\Delta P = R_m v(x) + \frac{n\phi RTrc_0}{D^2\gamma} v(x)^2 \int_0^x v(x')\,dx' \tag{16}$$

Equation 16 can be substantially simplified by rewriting in a dimensionless form. For this purpose, the following dimensionless quantities are introduced

$$\Delta F = \frac{\Delta P}{\Delta\pi_0} \tag{17}$$

$$V = v\left(\frac{L}{D^2\gamma}\right)^{1/3} \tag{18}$$

$$X = \frac{x}{L} \tag{19}$$

where L is the length (m) of the filtration channel. Replacing the original variables in Eq. 16 with the dimensionless quantities and rearranging yield

$$\Delta F = \left[\frac{R_m}{\Delta\pi_0}\left(\frac{D^2\gamma}{L}\right)^{1/3}\right] V + V^2 \int_0^X V\,dX' \tag{20}$$

The term in the square bracket is a dimensionless parameter that fully characterizes the cross-flow RO process. Let us denote this dimensionless parameter by N_s, that is

$$N_s = \frac{R_m}{\Delta\pi_0}\left(\frac{D^2\gamma}{L}\right)^{1/3} \tag{21}$$

Equation 20 becomes

$$\Delta F = N_s V + V^2 \int_0^X V\,dX' \tag{22}$$

N_s is the only parameter in Eq. 22. Comparing Eq. 22 with Eq. 16, it can be found that N_s is the membrane resistance in dimensionless form. The relationship between the permeate flux V and the driving force ΔF will be defined as long as N_s is given.

Analytical solution for permeate flux

An analytical solution of Eq. 22 can be obtained with a procedure developed before (Song and Elimelech, 1995). The solution procedure is briefly described below.

The integral in Eq. 20 vanishes at $X = 0$, and, therefore, we have

$$V(0) = \frac{\Delta F}{N_s} \tag{23}$$

For $X \neq 0$, it can be shown that the permeate flux satisfies the following differential equation

$$\frac{dV}{dX} = \frac{V^4}{N_s V - 2\Delta F} \tag{24}$$

Integrating Eq. 24 with Eq. 23 as a boundary condition results in

$$\left[\left(\frac{N_s}{\Delta F}\right)^3 + \frac{6X}{\Delta F}\right] V^3 + 3\frac{N_s}{\Delta F} V - 4 = 0 \tag{25}$$

The local permeate velocity $V(X)$ in the cross-flow RO channel can be found from Eq. 25 as

$$V(X) = \frac{\Delta F/N_s}{\left(1 + 6\Delta F^2 X/N_s^3\right)^{1/3}} \times \left\langle \left\{\left[\frac{1}{\left(1 + 6\Delta F^2 X/N_s^3\right)} + 4\right]^{1/2} + 2\right\}^{1/3} - \left\{\left[\frac{1}{\left(1 + 6\Delta F^2 X/N_s^3\right)} + 4\right]^{1/2} - 2\right\}^{1/3} \right\rangle \tag{26}$$

The average flux of the RO process $\bar{V}$ is then determined as

$$\bar{V} = \int_0^1 V(X)\,dX = \frac{\Delta F}{V(1)^2}\left[1 - \frac{N_s}{\Delta F} V(1)\right] \tag{27}$$

With Eq. 18, the local flux $V(X)$ and the average flux $\bar{V}$ can be transformed back to dimensional form $v(x)$ and $\bar{v}$.

The wall concentration of the retained solute is related to the local permeate flux by

$$C_w(x) = \frac{\Delta P - R_m v(x)}{n\phi RTr} \tag{28}$$

The polarization index I can be expressed as

$$I = \frac{c_w(x)}{c_0} = \frac{c_0 + C_w(x)}{c_0} = 1 + \frac{\Delta P - R_m v(x)}{\Delta\pi_0} = \frac{\Delta p - R_m v(x)}{\Delta\pi_0} \tag{29}$$

Simulations and Discussion

In this section, the local variations of permeate flux and polarization index along the filtration channel are first demonstrated to justify the effort made in this article over the simple assumption of a uniform concentration polarization layer. Then, the effects of various parameters on permeate flux in the cross-flow RO process are simulated and discussed with the newly developed theory. In the discussions, solutions of a single univalent electrolyte NaCl are referred to. Complete salt rejection is assumed for membranes with constant pressures. When the effect of pressure on permeate flux is investigated, the rejection itself is a function of pressure that is determined with Eq. 4. The osmotic coefficient ϕ is assumed to be unity throughout this section.

Local variation of permeate flux and polarization index

The concentration polarization layer is treated as a uniform layer along the cross-flow RO filtration channel in previous models. The validity of this treatment is checked with the new theory developed in this article. Figure 1 gives the local permeate fluxes along the channel for different feed concentrations in a cross-flow RO process. It can be seen that the fluxes for all feed concentrations decrease along the channel, even for the feed concentration as low as 0.03 M. It suggests that the local variations of concentration polarization have to be considered in most RO processes in order to obtain accurate predictions of the permeate fluxes. Figure 1 also shows that the fluxes drop faster in the initial segment of the channel and the declining rates become smaller downstream along the filtration channel. This observation tells us that the concentration polarization layer develops quickly in the channel. After the concentration polarization layer has significantly developed, it reduces permeate flux and, as a result, the further development of concentration polarization will become slower. Comparing the local variations of the fluxes for different feed concentrations, we can find that the concentration polarization layer is more significant for high feed concentrations. The reason is that a larger amount of salt is available for the development of the concentration polarization layer when the feed concentration is higher.

Wall concentration or the equivalent [polarization index (c_w/c_0)], is an important indicator of RO process operation because some sparing soluble solutes, such as calcium sulfate and carbonate, may precipitate under high concentration polarization. The local polarization indexes (c_w/c_0) along the channel for the same conditions as in Figure 1 are predicted and shown in Figure 2. From Figure 2, we can see that the

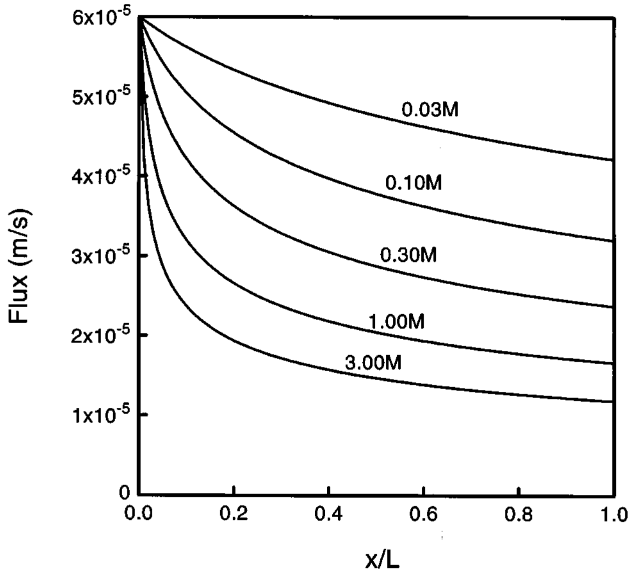

Figure 1. Variations of local permeate fluxes along a cross-flow RO channel for different feed concentrations of sodium chloride solutions.

Parameters used in the simulations are: $\Delta P = 3.0 \times 10^7$ Pa; $R_m = 5.0 \times 10^{11}$ Pa·s/m; $\gamma = 1{,}000$ s^{-1}; $L = 1.0$ m; $T = 298$ K.

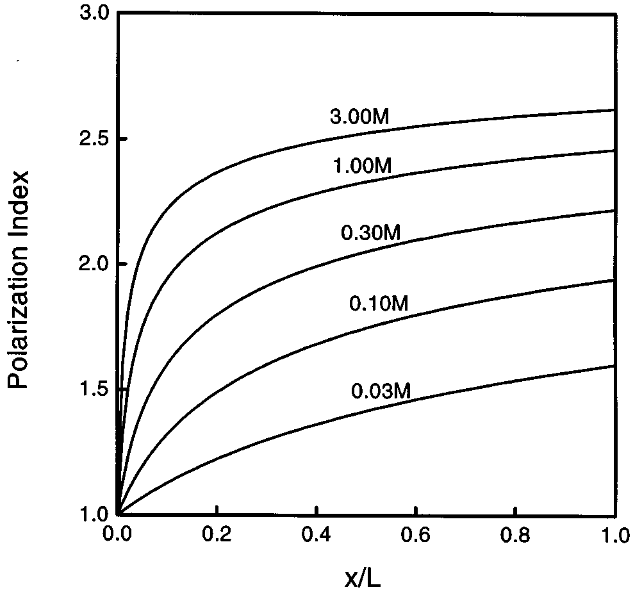

Figure 2. Variations of concentration polarization factors (c_w/c_0) along a cross-flow RO channel for different feed concentrations of sodium chloride solutions.

Parameters used in the simulations are: $\Delta P = 3.0 \times 10^7$ Pa; $R_m = 5.0 \times 10^{11}$ Pa·s/m; $\gamma = 1{,}000$ s^{-1}; $L = 1.0$ m; $T = 298$ K.

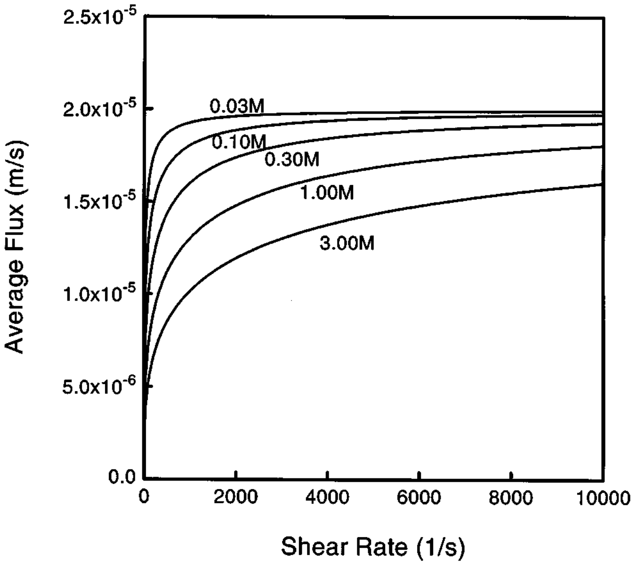

Figure 3. Effect of shear rate on average permeate fluxes of a cross-flow RO process for different feed concentrations of sodium chloride solutions.

Parameters used in the simulations are: $\Delta P = 3.0 \times 10^7$ Pa; $R_m = 5.0 \times 10^{11}$ Pa·s/m; $L = 1.0$ m; $T = 298$ K.

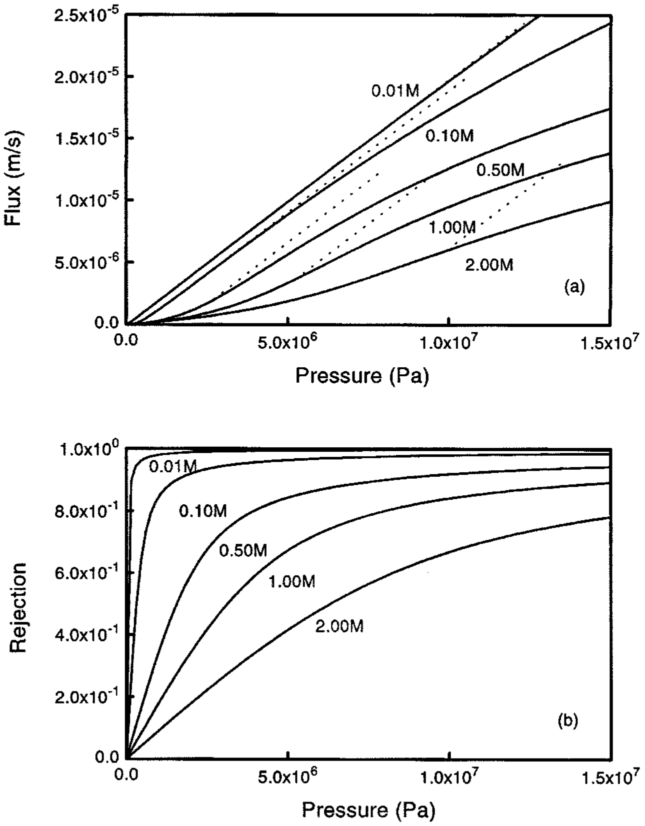

Figure 4. Effect of applied pressure on (a) permeate fluxes and (b) salt rejections in a cross-flow RO process with membrane selectivity $S = 10$.

Other parameters used in the simulations are: $R_m = 5.0 \times 10^{11}$ Pa·s/m; $\gamma = 1{,}000$ s^{-1}; $L = 1.0$ m; $T = 298$ K.

wall concentrations increase along the flow channel and reach their maximum value at the end of the channel. A similar result was obtained by Bhattacharyya et al. (1990) with a numerical solution of the convection-diffusion equation. It is also observed in Figure 2 that the polarization index is strongly affected by the feed salt concentration. Higher feed concentration tends to induce more significant concentration polarization.

Concentration polarization layer in the cross-flow reverse osmosis process is characterized by significant local variation. The assumption of a uniform concentration polarization layer oversimplifies the cross-flow reverse osmosis process and should not be used.

Effect of shear rate on the average flux

Feed flow rate is another important operating parameter in the cross-flow RO process. Permeate flux increases with increasing feed flow rate because of the depression of the concentration polarization layer. On the other hand, energy consumption also increases for higher feed flow rate due to the reduced recovery rate of the system. A better understanding of the effect of shear rate on the average permeate flux of a RO system can help us to run the system more efficiently. As a demonstration, simulations of the average fluxes as functions of shear rate for different feed concentrations are plotted in Figure 3. From this figure, we can see that the average fluxes generally increase with increasing shear rate. The increments of the average fluxes become smaller as the shear rate increases.

The effect of shear rate on the average flux is strongly affected by feed concentration. The rise of shear rate can result in substantial increase of permeate production for high feed concentrations, but it might only have a marginal effect on permeate flux for processes with feeds of very low concentrations. As shown in Figure 3, when the feed concentration is 0.03 M, the average permeate flux almost reaches its maximum value at a shear rate as flow as 1,000 s^{-1}. Further increase in shear rate beyond 1,000 s^{-1} does not have any notable effect on permeate flux. On the other hand, the average flux for feed concentration of 3.00 M increases in the whole range of shear rate shown in this figure.

Effect of pressure on the average flux

The dependence of permeate flux on pressure is much more complicated than on other parameters, such as shear rate and feed concentration. The complexity mostly comes from the variation of salt rejection with pressure. Therefore, in the investigation of the effect of pressure on the average permeate flux, the salt rejection at every pressure needs to be determined first. Equation 4 can be used to calculate salt rejection for a given membrane under any pressure when the selectivity S of the membrane is known.

The average permeate fluxes for different feed concentrations are simulated for a membrane with $S = 10$ and plotted (solid lines) in Figure 4a as functions of applied pressure (Δp). Also in Figure 4a are fluxes without considering concentration polarization (dotted lines). Corresponding salt re-

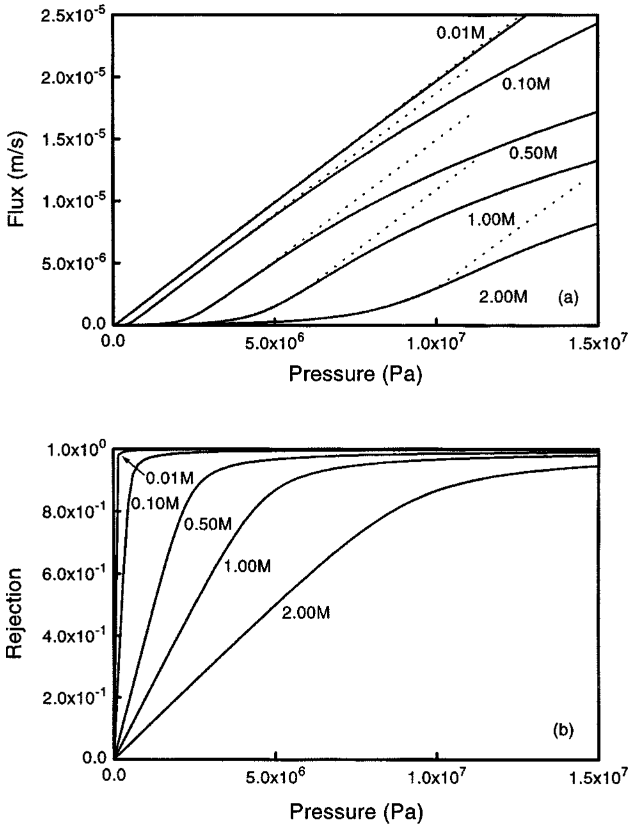

Figure 5. Effect of applied pressure on (a) permeate fluxes and (b) salt rejections in a cross-flow RO process with membrane selectivity $S=100$.

Other parameters used in the simulations are: $R_m = 5.0\times 10^{11}$ Pa·s/m; $\gamma = 1{,}000\ s^{-1}$; $L = 1.0$ m; $T = 298$ K.

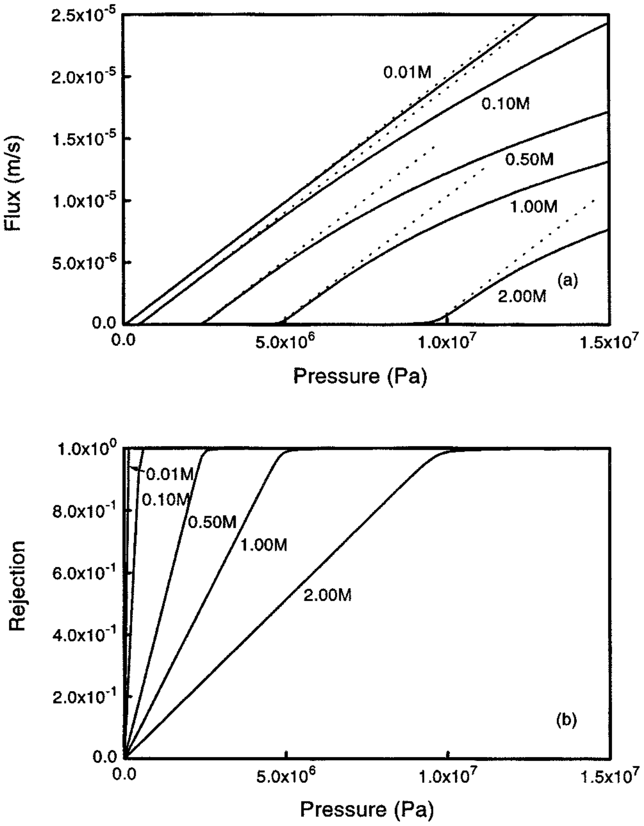

Figure 6. Effect of applied pressure on (a) permeate fluxes and (b) salt rejections in a cross-flow RO process with membrane selectivity $S=10^4$.

Other parameters used in the simulations are: $R_m = 5.0\times 10^{11}$ Pa·s/m; $\gamma = 1{,}000\ s^{-1}$; $L = 1.0$ m; $T = 298$ K.

jections are given in Figure 4b. It is seen from Figure 4a that there are no differences between the two groups of fluxes (with and without considering concentration polarization) under low pressures. This observation shows that concentration polarization is insignificant at low pressures and the flux curves is mainly determined by the membrane transport properties (Song, 1999). At high pressures, permeate fluxes increase linearly with pressure when concentration polarization is not considered (as shown with the dotted lines). The permeate fluxes under the influence of concentration polarization increase with pressure much slower than these without considering concentration polarization, especially for solutions of high salt concentrations.

Simulations of permeate fluxes under similar conditions are also conducted for other two membranes with $S=100$ and 10^4, respectively. The results are plotted with corresponding salt rejections in Figures 5 and 6. Figure 5a shows similar dependence of fluxes on pressure but with lower fluxes for high feed concentrations. The lower fluxes are due to the larger amount of salt rejected by the membrane with higher selectivity. The resultant additional osmotic pressure then further reduces the effective driving force. Figures 6a and 6b show different pattern of fluxes and rejections for membrane with very high selectivity from those of low selectivity. The rejections increase almost linearly from 0 to 1 as pressure Δp increases from 0 to π_0 ($= nRTc_0$). The permeate fluxes are practically zero for applied pressures smaller than the osmotic pressure of the solutions. There are appreciable permeate fluxes only when the applied pressures are larger than the solution osmotic pressures. The downward curvature of the flux lines in Figure 6a shows the effect of concentration polarization on the fluxes.

Conclusion

In the cross-flow RO process, concentration polarization develops gradually along the filtration channel as a result of the accumulation of the retained solute. Concentration polarization and its effect on permeate flux in the cross-flow RO process can be better understood and modeled when this local variation of the concentration polarization layer is properly considered.

The coupling between concentration polarization and permeate flux makes it very difficult to investigate concentration polarization or permeate flux separately. The theoretical effort in decoupling between flux and concentration polarization has long been unsatisfactory. A new attempt to solve this coupling has been made in this study. One important result of this decoupling is the dimensionless general equation for the cross-flow RO process. In this equation, the properties of

RO processes are characterized with a single dimensionless parameter N_s. When N_s is known, the relationship between permeate flux and applied pressure can be given in a closed-form analytical solution.

As demonstrated, the local permeate flux and polarization factor, the effect of various parameters on the average flux, and the relationship between permeate flux and applied pressure can be conveniently investigated with the new model. This model will find even more applications in process design, operation optimization, and fouling control.

Notation

c = solute concentration, mol/m^3
c_i = concentration of electrolyte i, mol/m^3
c_w = wall solute concentration, mol/m^3
C = concentration of the retained solute, mol/m^3
C_w = wall concentration of the retained solute, mol/m^3
v [or $v(x)$] = local permeate flux, m/s
X = dimensionless distance from the entrance section, defined by Eq. 19

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